# Chapter 3

## Nonlinear Motion

# Projectile Motion: 2-D Motion

- A projectile is any object that is projected by some means and continues in motion by it own inertia. (e.g. Balls, missiles)
- The velocity of a projectile has a horizontal and vertical component.
  - There is a velocity acting in the horizontal direction.
  - There is a velocity acting in a vertical direction.

# Projectile Motion: 2-D Motion

- Will look at 2 types of projectile motion
  - Launched with only horizontal velocity
    - Launched horizontally off cliff
    - Like Free Fall dropped from height except add $\mathbf{v}_h$
    - E.g. Pitched baseball
  - Launched with horizontal and vertical velocity
    - Cannon ball, basketball
    - Like Free Fall with initial vertical velocity (Ch2) but also has initial horizontal velocity

# Projectile Motion: Launched with horizontal velocity

## Vertical Motion

- Even though no initial vertical velocity caused by launching, there still is a vertical velocity caused by gravity. $\mathbf{a_v} = \mathbf{g} = 9.81 m/s^2$
- Vertical motion of the projectile is Free Fall.
- The vertical distance traveled each second increases (because motion is accelerating).

# Projectile Motion- Launched with only Horizontal Velocity

<u>Horizontal motion</u>

- At launch, a force is applied that gives the projectile a horizontal velocity.
- Once projectile is launched, there is <u>no</u> additional force that can change the velocity. Therefore $\mathbf{v_h}$ **= constant**
  - Once you throw a basketball and it has left your hands, you can't change how fast it goes

# Projectile Motion- Launched with only Horizontal Velocity

<u>Compare the distance traveled</u>

- Vertical distance
  - Vertical distance traveled each second increases.
  - Expected because in Free Fall.
  - Vertical distance traveled is unaffected by horizontal motion.
- Horizontal distance
  - Horizontal distance traveled each second is the same.
  - Expected because constant velocity.
  - Horizontal distance traveled is unaffected by vertical motion

# Projectile Motion- Launched with only Horizontal Velocity

<u>Summary</u>

- Motion is combination of two motion
  - Vertical motion is Free Fall
    - Constant vertical acceleration = g
    - Vertical distance traveled per second increases
  - Horizontal motion has constant velocity
    - No acceleration
    - Horizontal distance traveled per second is the same.

# Projectile Motion- Launched with only Horizontal Velocity

<u>Conclusion</u>

- The horizontal motion does not affected by the vertical motion
- The vertical motion is not affected by the horizontal motion

***The horizontal and vertical motions are independent of each other***

# Projectile Motion- Compare Free Fall with Horizontal Launch

- Recall that the horizontal motion does not affected by the vertical motion
- Recall that Free fall and Horizontal projectiles have same vertical motion

***The time for both objects to reach the ground is the same. Time to reach ground is only affected by vertical motion.***

# Projectile Motion- Launched Upward (vertical and horizontal velocity)

*Now let's look at the second type of projectile motion:* ***Launched upward***

- Projectile has been launched with a velocity at an angle
  - Has vertical velocity component
  - Has horizontal velocity component
- Path is a parabola Fig 3.10 – 3.15

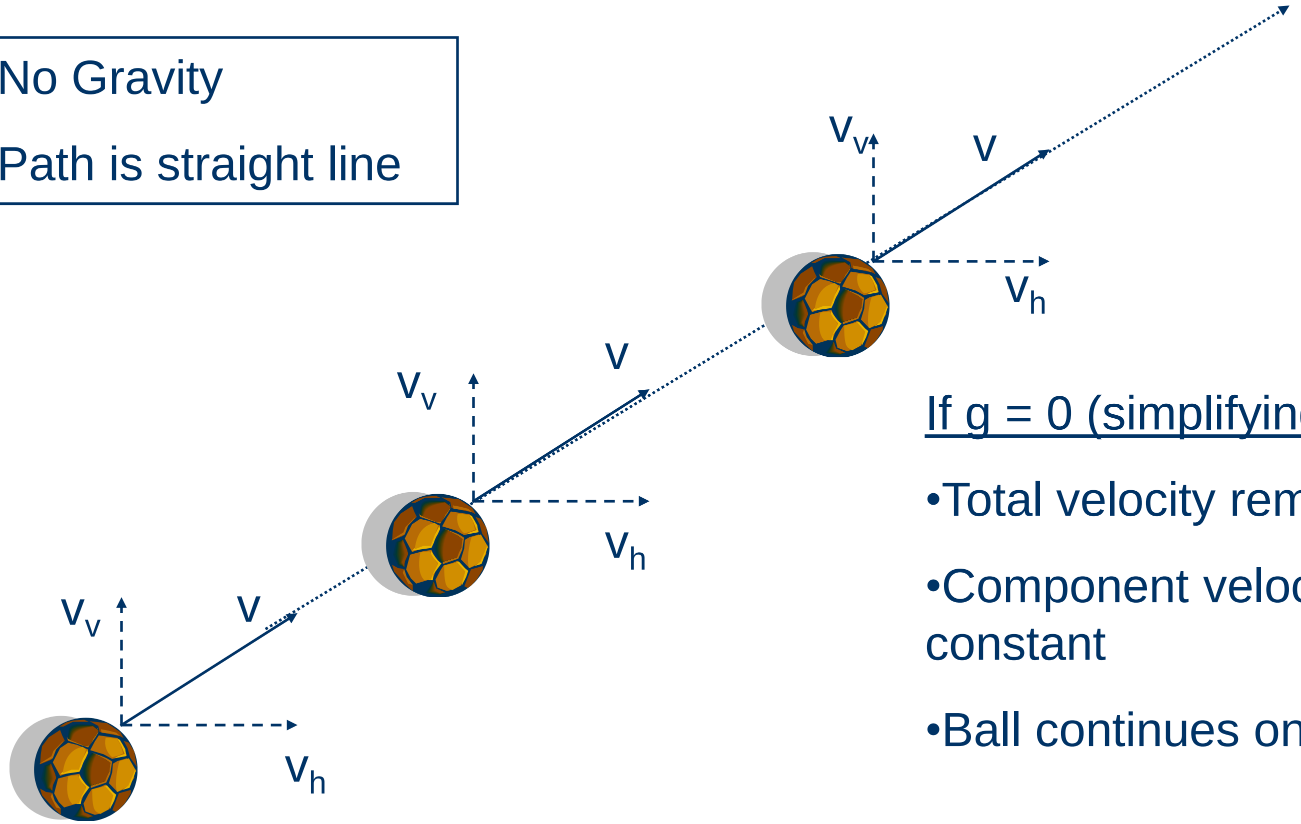
# Projectile Motion- Launched Upward (vertical and horizontal velocity)

- Let's start by simplifying the motion.
- Let's look at the projectile's motion and assume there is no gravity.
  - Have a constant velocity applies at an angle
  - No acceleration
  - Can resolve the velocity into its components

# Projectile Motion- Launched Upward (vertical and horizontal velocity)

No Gravity

Path is straight line




<u>If g = 0 (simplifying assumption)</u>

•Total velocity remains constant

•Component velocities remain constant

•Ball continues on straight path

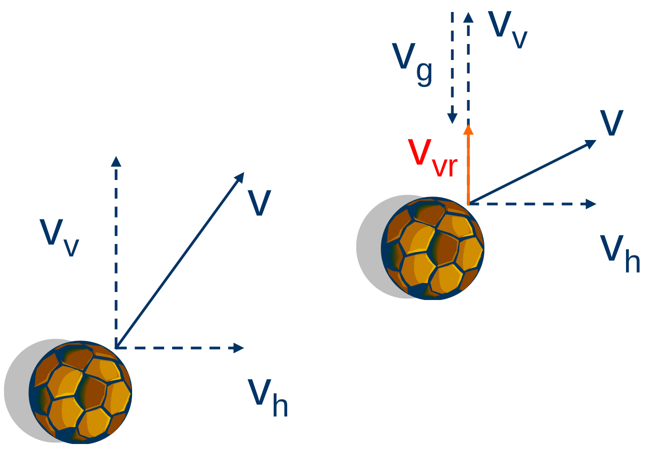
# Projectile Motion- Launched Upward (vertical and horizontal velocity)

- Now let's consider what happens when we add gravity
  - g act in vertical direction
  - g will not impact horizontal direction

# Projectile Motion- Launched Upward (vertical and horizontal velocity)

Add Gravity

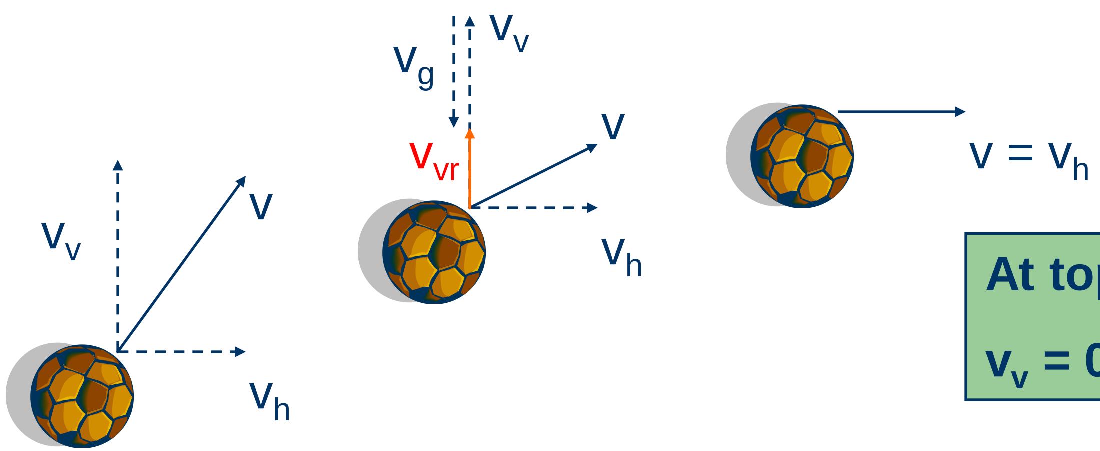
- $V_v$ decrease by gt




*Why has the total velocity, v, changed?*

# Projectile Motion- Launched Upward (vertical and horizontal velocity)

- Eventually, the resultant vertical velocity will decease to zero: $v_{vr} = 0$
- The total velocity, v, will equal just the horizontal velocity: $v = v_h$
- The ball still has an acceleration acting on it→ g

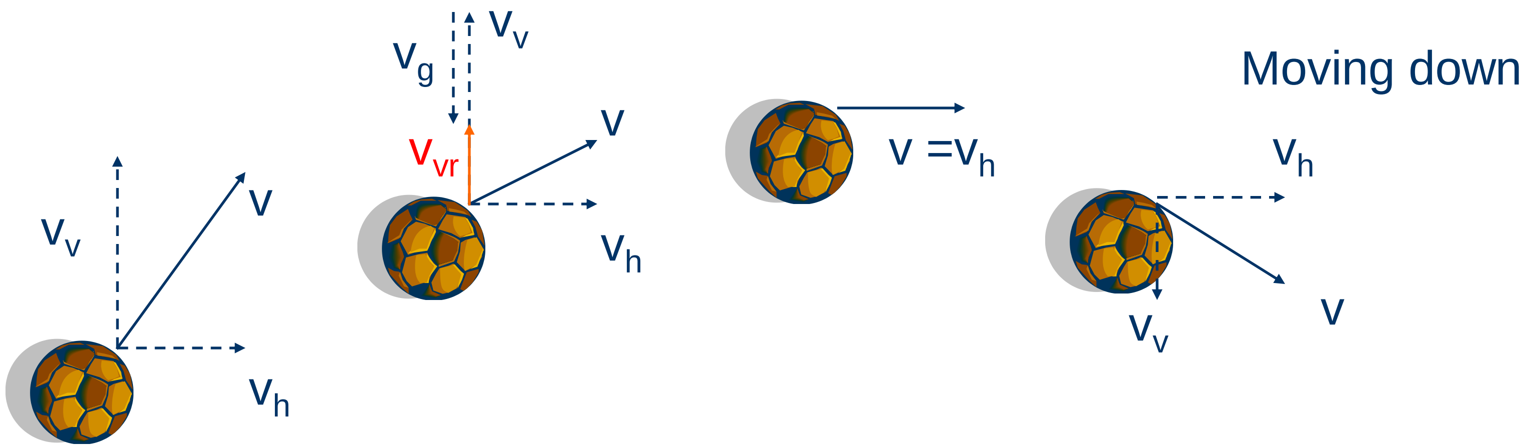



**At top of path**

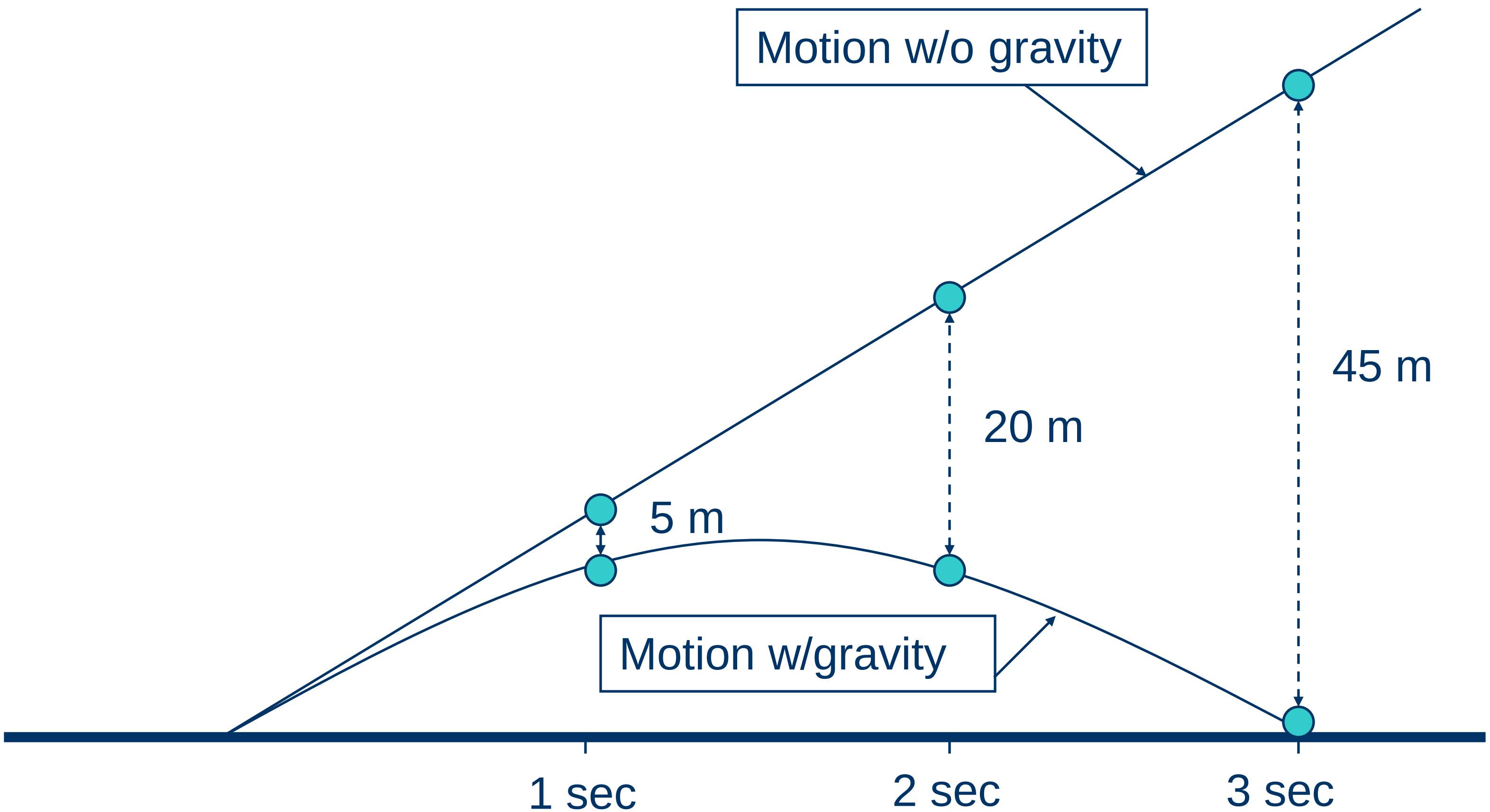
**$v_v = 0$ but $v \neq 0$ (equals $v_h$)**

# Projectile Motion- Launched Upward (vertical and horizontal velocity)

- The vertical velocity is now downward because gravity pulling on ball
- The total velocity, v, still sum of components and heading downward
- Throughout path, $v_h$ has not changed



# Projectile Motion- Comparing Straight Line and Parabolic Projectile Motion



# Projectile Motion- Comparing Straight Line and Parabolic Projectile Motion

The vertical distance between the projectile's straight line (no gravity) path and parabolic (w/gravity) path is

$$d = \frac{1}{2} g t^2$$

Equation for distance with constant acceleration (Ch 2)

# Projectile Motion: Launch Angle

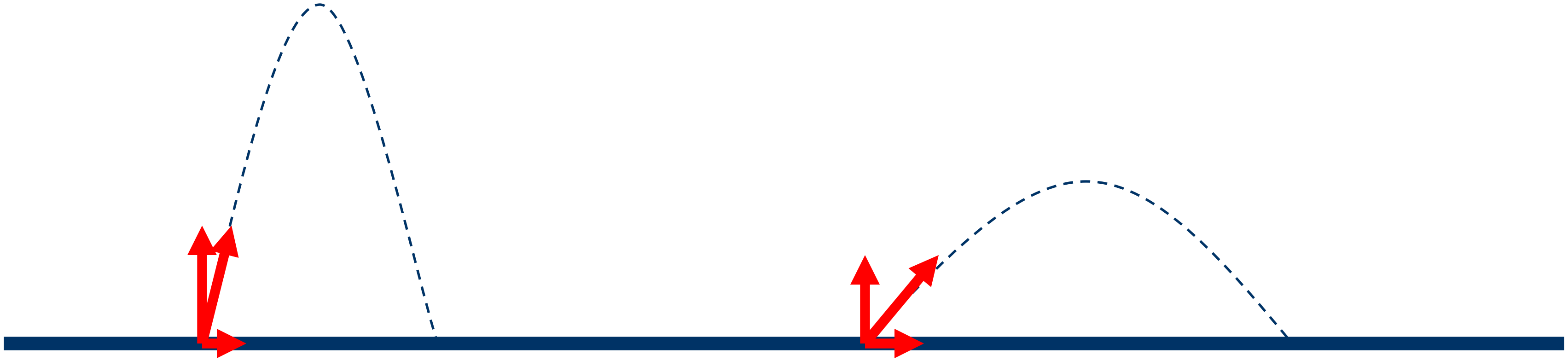
## How do different launch angles impact projectile?.

The steeper the launch angle:

- The greater the initial $v_v$
- The steeper the path
- The higher the total height

The shallower the launch angle:

- The lower the initial $v_v$
- The flatter the path
- The lower the total height



# Projectile Motion: Range

- Range is the horizontal distance traveled
- The greater the initial $v_h$, the longer the <u>range.</u>
- Two projectiles will have <u>same range</u> if their launch angles add up to 90°.
- <u>Maximum range</u> for a projectile is achieved with a projection angle of 45°.
- Figure 3.13

# Projectile Motion : Air Resistance

- For low speed projectiles, often assume air resistance is zero.
- For high-speed projectiles, air resistance cannot always assume air resistance is zero
- For high-speed projectiles, air resistance causes the trajectory to fall short of ideal (no air resistance) parabolic path.
- Figure 3.15

# Fast-Moving Projectiles: Satellites

- An earth satellite is simply a projectile that falls around the earth rather that into it.
- The speed of the satellite must be great enough to ensure that its falling distance matches the earth's curvature.
- It takes 1 second for an object in free fall to fall 5 meters.
- Earth's surface 'drops' a vertical distance of 5 meters for every 8000 meters along the Earth's surface.

# Fast-Moving Projectiles: Satellites

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# Fast-Moving Projectiles: Satellites

- This means that a satellite near the Earth's surface must travel at <u>8000 meters/second</u>!
    - …or 18,000 miles per hour.
- For example:  The space shuttle orbits the Earth once every 90 minutes.

# 2-Dimensional Motion

***End of Chapter 3***