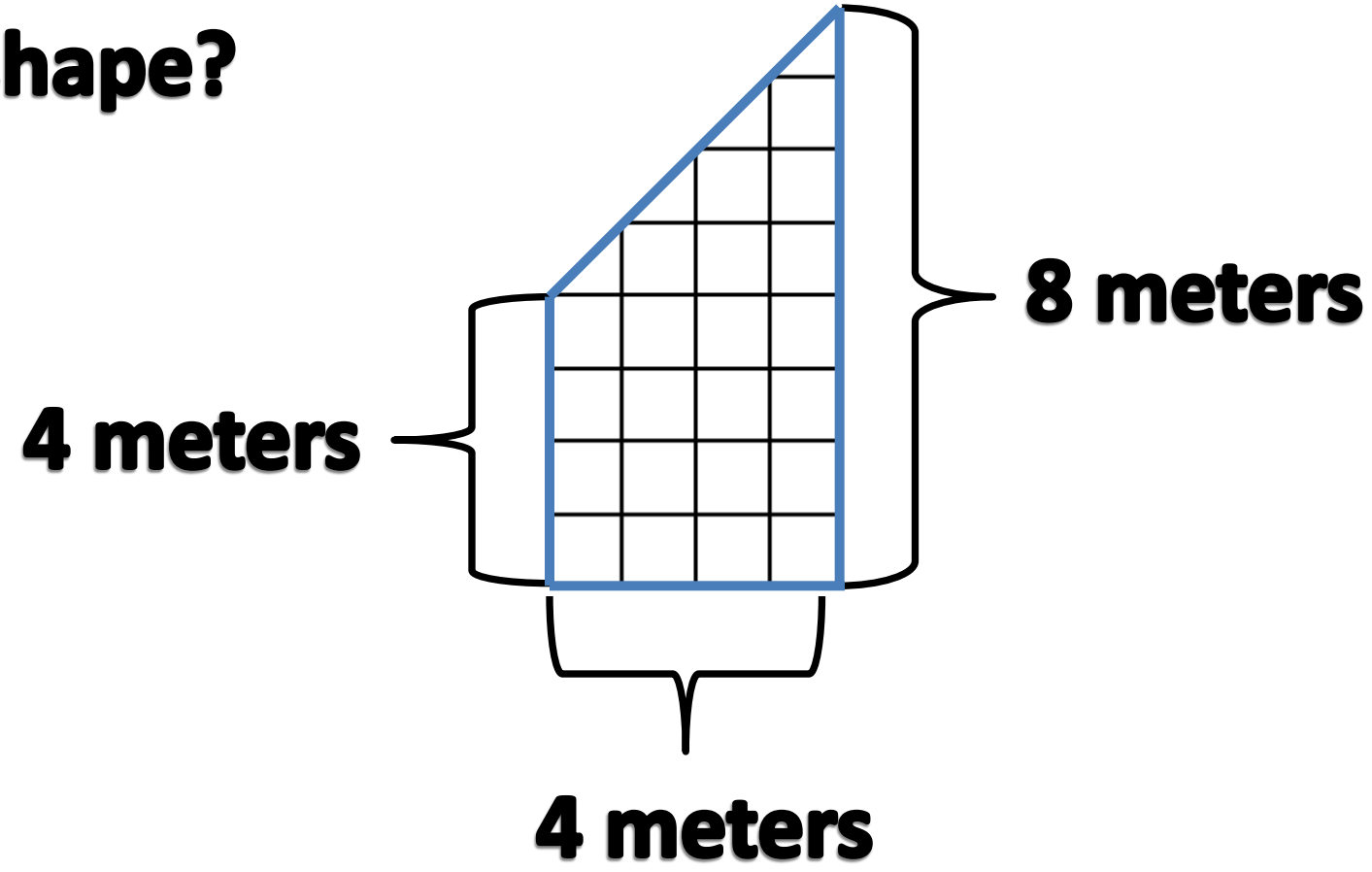
Deriving the Kinematic Equations

What is the area of this shape?



What is the formula for distance, given speed and time?

Find a simple algebraic formula for area of the shape using the terms of v_o, v_f and t.

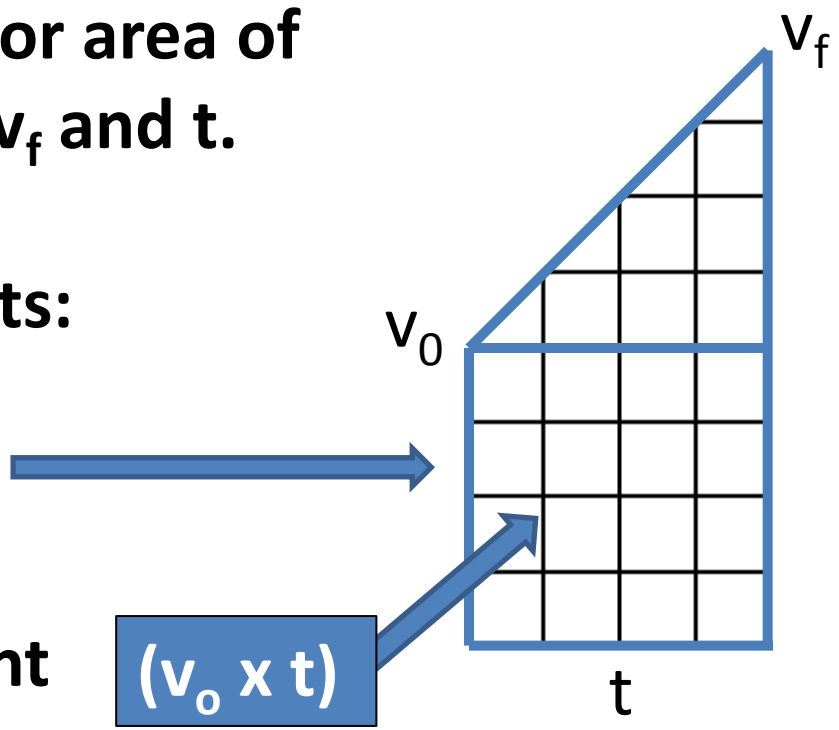
Break this problem in to two parts:

Find the area of the rectangle?

Area of a rectangle= Base x Height

So the area would be….

$$(v_o \times t)$$



Notice that v = d/t

So… d = v x t

This means when you find the area of a speed verses time graph you are finding the distance it traveled!

Find the area of the triangle?

The area of a triangle = ½ x Base x Height

The Height of the triangle would simple be the difference between V_f and V_o. So the formula for the area of the triangle written out would look like:

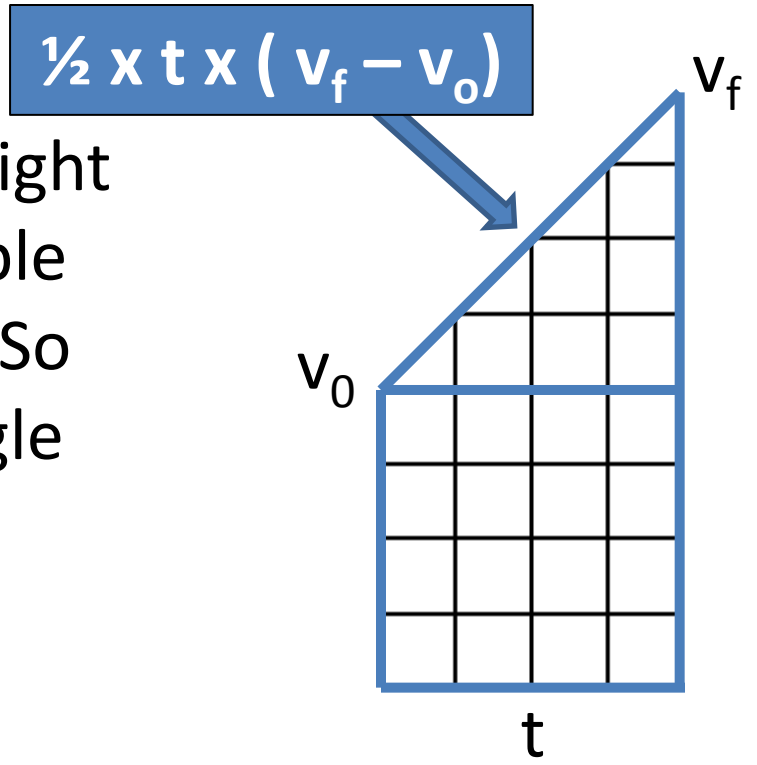
$$\frac{1}{2} \times t \times (v_f - v_o)$$

or

$$\frac{1}{2} t (v_f - v_o)$$

Notice by distributing ½ and t, you can rearrange the equation to look like this:

$$\left(\frac{1}{2} v_f t\right) - \left(\frac{1}{2} v_o t\right)$$



Notice that v = d/t

So… d = v x t

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Area of the rectangle:

$(v_o \times t)$

Area of the triangle:

$(\frac{1}{2} v_f t) - (\frac{1}{2} v_o t)$

Now you can add the area of the to the triangle area of the rectangle

$$\Delta d = (\tfrac{1}{2} v_f t) - (\tfrac{1}{2} v_o t) + (v_o t)$$

Combining Like terms:

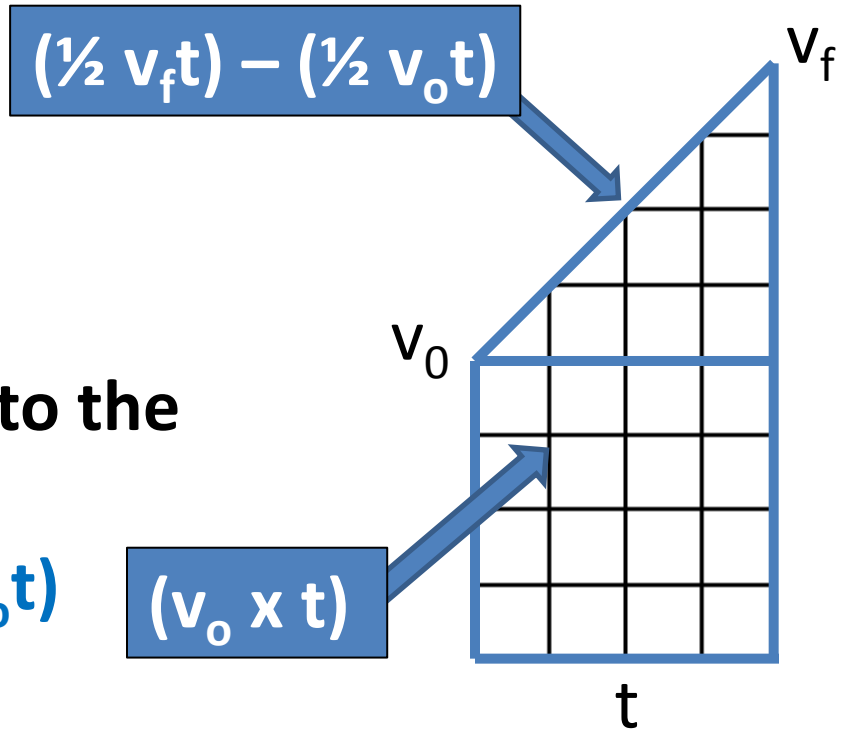
$$\Delta d = (\tfrac{1}{2} v_f t) - (\tfrac{1}{2} v_o t) + (v_o t) = (\tfrac{1}{2} v_f t) + (\tfrac{1}{2} v_o t)$$

$-\frac{1}{2} + 1 = +\frac{1}{2}$

$$\Delta d = (\tfrac{1}{2} v_f t) + (\tfrac{1}{2} v_o t)$$

Rearrange again: (pull out ½ and t)

$$\Delta d = \tfrac{1}{2} (v_o + v_f) t$$



Notice that v = d/t

So… d = v x t

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The Kinematic Equations

Remember the area under a Velocity vs. Time graph is distance.

1st Kinematic Equation

$$\Delta d = \frac{1}{2}(v_i + v_f)t$$

Notice "a" is not used!

Found from Slope of a Velocity vs. Time Graph.

2nd Kinematic Equation

$$v_f = v_i + (at)$$

Notice "Δd" is not used!

Substituting v_f in the 1st kinematic equation with the 2nd equation (standard form).

3rd Kinematic Equation

$$\Delta d = (v_i t) + (\frac{1}{2} a(t^2))$$

Notice "v_f" is not used!

Solve the second kinematic equation for v_i. Substitute this for vi in to the 1st will derive the following.

4th Kinematic Equation

$$\Delta d = (v_f t) - (\frac{1}{2} a(t^2))$$

Notice "v_i" is not used!

Solving the 1st equation for t, then substituting this into the second equation for t we find the following.

5th Kinematic Equation

$$v_f^2 = v_i^2 + (2a\,\Delta d)$$

Notice "t" is not used!